

# Quantum chaos in many-particle systems

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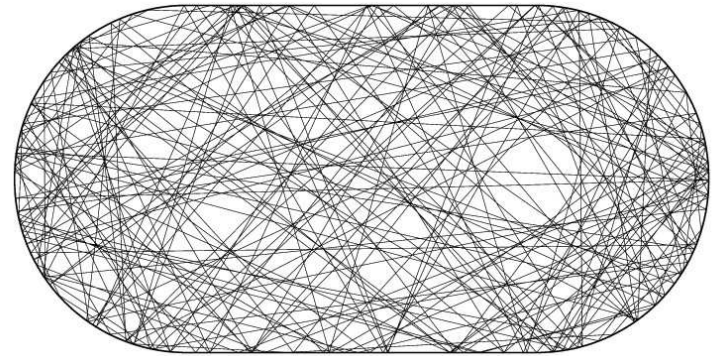
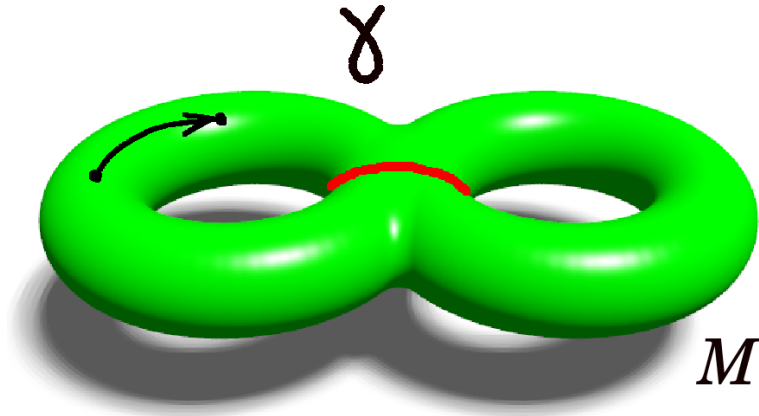
# Outline of the talk

- “Single”-particle quantum chaos.  
Single (semiclassical) limit:  $\hbar \rightarrow 0$

- Many-particle quantum chaos.  
Double limit:  $N \rightarrow \infty, \hbar \rightarrow 0$

B.G. & V. Al. Osipov, *Nonlinearity* 29 (2016)  
arXiv:1503.02676

# Chaos & Spectral universality



**Classical chaos:**  $\delta(t) \sim \delta(0)e^{\lambda t}$

## Motivation

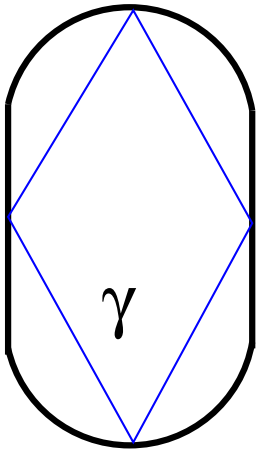
**Quantum:**  $-\Delta\varphi_n = \lambda_n\varphi_n, \quad \varphi_n \in L^2(M)$

**BGS conjecture** G.Casati, et al. 1980; O. Bohigas, et al. 1984: Correlations of  $\{\lambda_n\}_{n=1}^{\infty}$  are universal, described by Random Matrix Ensembles from the same symmetry class

# Semiclassical approach

Gutzwiller's trace formula:

$$\rho(E) = \sum_n \delta(E - E_n) \sim \underbrace{\bar{\rho}(E)}_{\text{Smooth}} + \underbrace{\Re \sum_{\gamma \in \text{PO}} \mathcal{A}_\gamma \exp\left(\frac{i}{\hbar} S_\gamma(E)\right)}_{\text{Oscillating}}$$



$\mathcal{A}_\gamma$  stability factor,  
 $S_\gamma(E)$  action of a **periodic orbit**  $\gamma$

**Number of periodic orbits grows exponentially** with length

- No prediction on  $E_n$  from an individual  $\gamma$
- **All  $\{\gamma\}$  together  $\iff$  spectrum**

# Two-point correlation function

$$R(\varepsilon) = \frac{1}{\bar{\rho}^2} \langle \rho(E + \varepsilon/\bar{\rho}) \rho(E) \rangle_E - 1$$

$$K(\tau) = \int_{-\infty}^{+\infty} R(\varepsilon) e^{-2\pi i \tau \varepsilon} d\varepsilon \approx \text{(Semiclassically)}$$

$$\approx \frac{1}{T_H^2} \left\langle \sum_{\gamma, \gamma'} \mathcal{A}_\gamma \mathcal{A}_{\gamma'}^* e^{\frac{i}{\hbar}(S_\gamma - S_{\gamma'})} \delta \left( \tau - \frac{(T_\gamma + T_{\gamma'})}{2T_H} \right) \right\rangle_E,$$

$T_\gamma, T_{\gamma'}$  are periods of  $\gamma, \gamma'$ ,  $T_H = 2\pi\hbar\bar{\rho}$  (Heisenberg time)

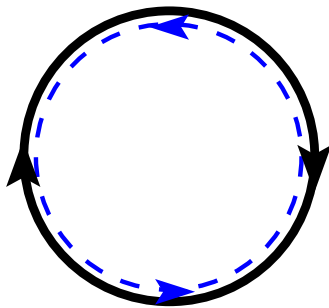
**Spectral correlations**  $\iff$

**Correlations between actions of periodic orbits**

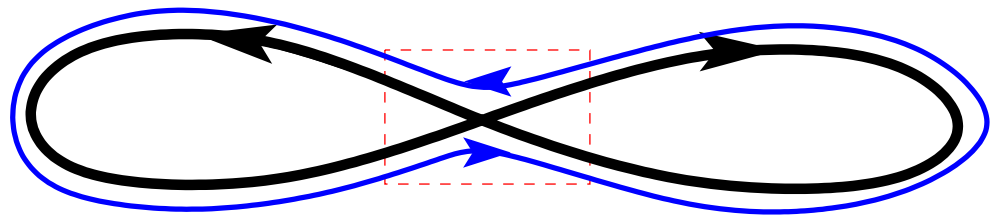
# Classical origins of universality

$$K(\tau) = c_1 \tau + c_2 \tau^2 \dots$$

$c_1$  – diagonal approximation  $\gamma = \gamma'$  M. Berry 1985



Diagonal approximation



Sieber–Richter pairs

$c_2$  – non-trivial correlations (Sieber-Richter pairs)  
M. Sieber K. Richter 2001

$S_\gamma - S_{\gamma'} \sim \hbar \implies \text{Duration of encounter} \sim \underbrace{\tau_E = \lambda^{-1} |\log \hbar|}_{\text{Ehrenfest time}}$

All orders in  $\tau$  = **RMT result** S. Müller, et. al., 2004

# Symbolic Dynamics

Continues flow  $\implies$  Map  $T$  (Poincare section)

$\mathbf{p}$

$0$	$1$
$\vdots$	$\vdots$
$l-2$	$l-1$

$\mathbf{q}$

**Phase space partition:**

$$V = V_0 \cup V_1 \cup \dots \cup V_{l-1}$$

**Point in the phase space:**

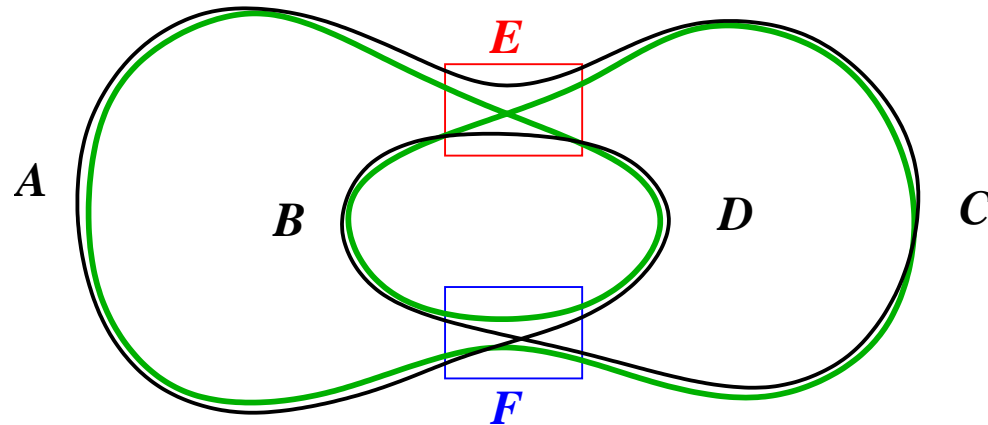
$$x = \underbrace{\dots x_{-1}x_0}_{\text{past}} \cdot \underbrace{x_1x_2\dots}_{\text{future}}; \quad x_i \in \underbrace{\{0, 1, \dots, l-1\}}_{\text{alphabet}}$$

$$Tx = \dots x_{-1}x_0x_1 \cdot x_2x_3 \dots$$

**Periodic orbits**  $\iff [x_1x_2 \dots x_n]$

# Partner orbits

B. G, V. Osipov 2013



$$[\gamma_1] = [AECFBEDF], \quad [\gamma_2] = [AEDFBECECF]$$

$$E = e_1 e_2 \dots e_p, \quad F = f_1 f_2 \dots f_p$$

Each p-subsequence of symbols from  $\gamma_1$  appears in  $\gamma_2$   
**Locally similar but not identical**  $\Rightarrow$

Two orbits pass approximately the same points of the phase space:

$$\|\gamma_1 - \gamma_2\| \sim \Lambda^{-p}$$



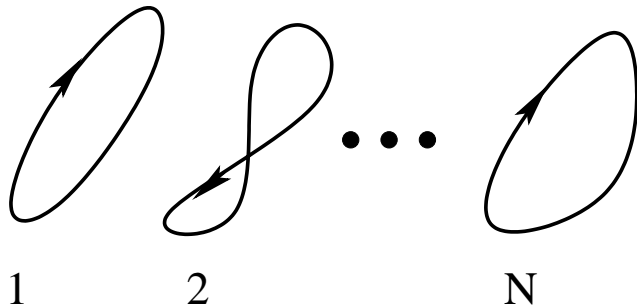
# Many-particle systems

$$\mathcal{H} = \sum_{n=1}^N \frac{p_n^2}{2m} + V(x_n) + V_{\text{int}}(x_n - x_{n+1})$$

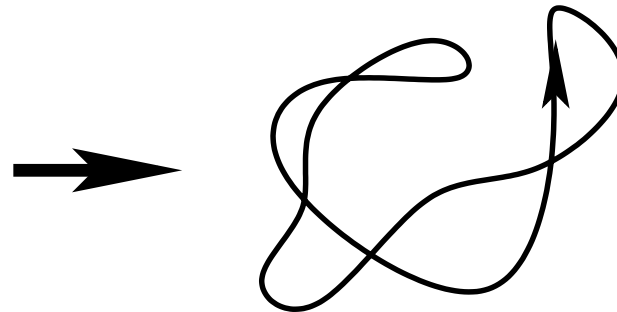
Chaos, Local interactions, Invariance under  $n \rightarrow n + 1$

## Two views on dynamics:

Many-particle Periodic Orbit  
**d-dimensions**



Single-particle Periodic Orbit  
**Nd-dimensions**



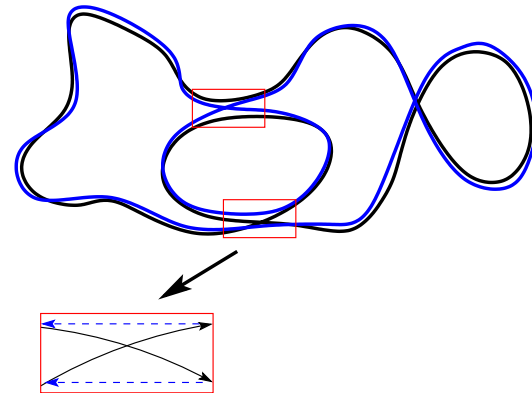
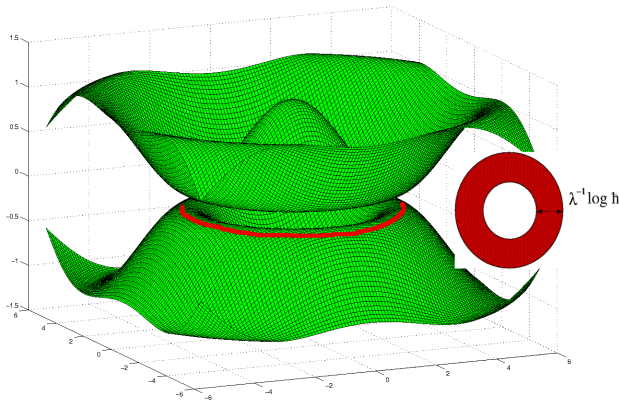
**Q: Is the single-particle theory of Quantum Chaos applicable?**

# Semiclassical “Field Theory”

**Continuous limit:**  $n \rightarrow \eta \in [0, \ell]$ ,  $x_{n,t} \rightarrow \phi(\eta, t)$

$$\mathcal{L} = \sum_{n=1}^N \frac{\dot{q}_{n,t}^2}{2m} + \kappa(x_{n,t} - x_{n+1,t})^2 - V(x_{n,t}) \implies$$

$$\mathcal{L} = \int_0^\ell d\eta \left( (\partial_t \phi(\eta, t))^2 + (\partial_\eta \phi(\eta, t))^2 - V(\phi(\eta, t)) \right)$$



- 1) **PO** -are **2D toric surfaces** in  $d$ -dim space (Rather than 1D lines in  $N \cdot d$ -dim)
- 2) **Encounters** are “**rings**” (Rather than 1D stretches) of “width”  $\sim \lambda^{-1} |\log \hbar_{eff}|$

# 2D Symbolic Dynamics

4
1
3
1
2
1
2
3
4
3
2
4
3
1
2
3
1

T

2	4	1	2	1	4	2	1	3	4	2	3	2	3	4	3
2	3	1	2	1	3	4	2	3	1	2	3	1	4	2	3
3	2	1	4	3	4	1	1	3	1	4	1	2	1	1	
2	4	2	1	4	2	4	3	3	1	2	4	4	1	2	3
4	1	4	3	2	4	2	1	4	4	1	4	3	4	1	2
3	4	3	1	3	2	4	2	3	1	2	1	4	2	4	4
1	1	2	4	3	1	3	2	3	1	4	3	3	4	2	4
1	4	2	3	1	4	2	4	4	4	3	4	2	2	4	3
3	2	3	1	4	1	4	1	2	1	2	4	3	1	3	1
1	4	2	2	3	2	1	3	4	4	2	3	1	4	2	3
3	1	1	4	1	4	3	2	1	3	2	2	3	3	1	4

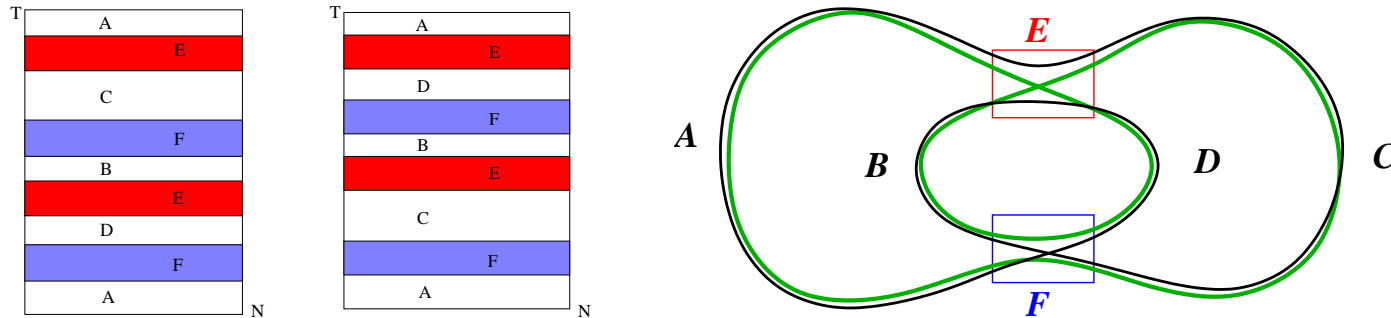
N

- 1) **Small alphabet** (does not grow with  $N$ )
- 2) **Uniqueness:** Each PO  $\Gamma$  is uniquely encoded by  $\mathbb{M}_\Gamma$
- 3) **Locality:**  $r \times r$  square of symbols around  $(n, t)$  defines position of the  $n$ 'th particle at the time  $t$  up to error  $\sim \Lambda^{-r}$

## Encounter - repeating region of symbols

# Different types of Partner Orbits

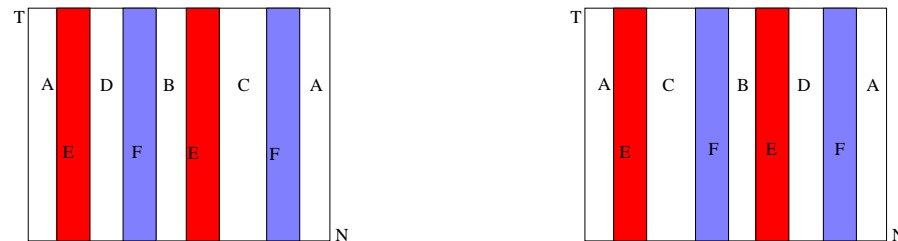
## A. Single particle partners:



Dominant iff  $T \gtrsim W_{\hbar} \gtrsim N$  - **Single particle theory**

$$W_{\hbar} \sim \Lambda^{-1} |\log \hbar_{eff}| \approx \text{Width of encounter}$$

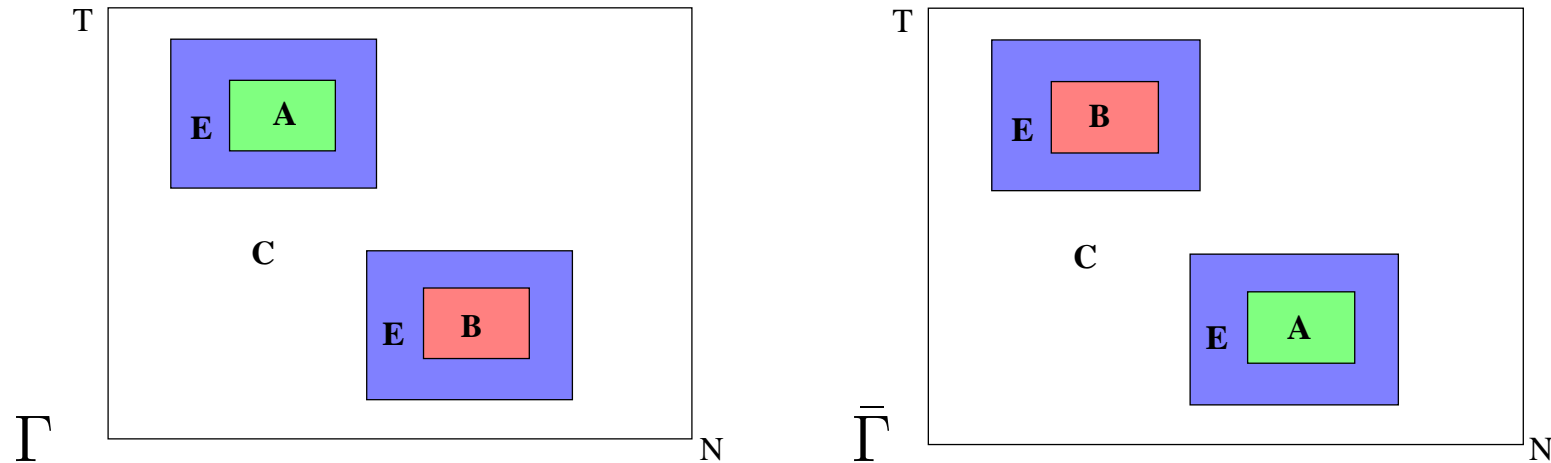
## B. Dual partners:



Dominant iff  $T \lesssim W_{\hbar} \lesssim N$  - **Thermodynamic, short time regime**

# Different types of Partner Orbits

**C.** If  $T \gtrsim W_{\hbar}$ ,  $N \gtrsim W_{\hbar}$  i.e.  $T$  and  $N$  are larger then  
“Ehrenfest scale”:

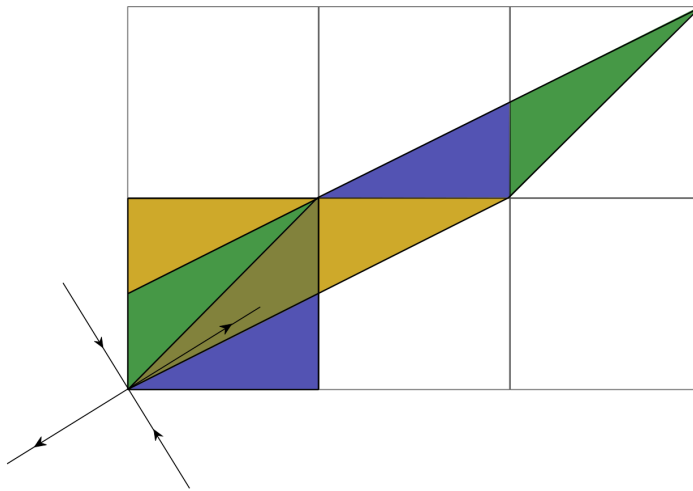


Note: One encounter is enough, even if time reversal symmetry is broken

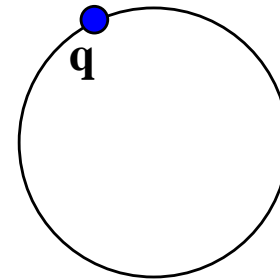
**B, C - Genuine many-particle Quantum Chaos!**

# A Lone Cat Map: $\mathbb{T}^2 \rightarrow \mathbb{T}^2$

**Phase space:**  $q_t, p_t \in [0, 1)$ , **windings**  $\mathbf{m}_t = (m_t^q, m_t^p) \in \mathbb{Z}$



*Configuration Space*

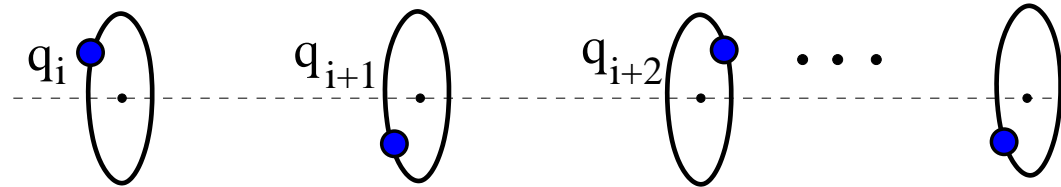


$$\begin{pmatrix} q_{t+1} \\ p_{t+1} \end{pmatrix} = \begin{pmatrix} a & 1 \\ ab - 1 & b \end{pmatrix} \begin{pmatrix} q_t \\ p_t \end{pmatrix} - \begin{pmatrix} m_t^q \\ m_t^p \end{pmatrix},$$

$a, b \in \mathbb{Z}$ . Chaos if  $|a + b| > 2$

**Newton form:**  $\Delta q_t \equiv q_{t+1} - 2q_t + q_{t-1} = (a + b - 2)q_t - m_t$

# Coupled-Cat Maps: $T^{2N} \rightarrow T^{2N}$



$$S(\mathbf{q}_t, \mathbf{q}_{t+1}) = S_0(\mathbf{q}_t, \mathbf{q}_{t+1}) + S_{\text{int}}(\mathbf{q}_t), \quad \mathbf{q}_t = (q_{1,t}, q_{2,t} \cdots q_{N,t})$$

$N$  Interacting cat maps,  $q_{n,t}, p_{n,t} \in [0, 1)$ :

$$S_0 = \sum_{n=1}^N S_{\text{cat}}(q_{n,t}, q_{n,t+1}) + V(q_{n,t}); \quad S_{\text{int}} = - \underbrace{\sum_{n=1}^N q_{n,t} q_{1+n,t}}_{\text{interactions}}$$

Equations of motion:

$$p_{n,t} = -\frac{\partial S}{\partial q_{n,t}} \quad p_{n,t+1} = \frac{\partial S}{\partial q_{n,t+1}}$$

# Classical Particle-time Duality

**Newtonian form:**

$$\Delta q_{n,t} = (a + b - 4)q_{n,t} + V'(q_{n,t}) - m_{n,t}$$

**Discrete Laplacian:**

$$\Delta f_{n,t} \equiv f_{n+1,t} + f_{n-1,t} + f_{n,t+1} + f_{n,t-1} - 4f_{n,t}$$

**Particle-time symmetry:**  $t \longleftrightarrow n \implies$

**$N$ -particle POs  $\{\Gamma\}$  of period  $T \iff T$ -particle POs  $\{\Gamma'\}$   
of period  $N$**

$$S(\Gamma) = S(\Gamma'), \quad A_\Gamma = A_{\Gamma'}$$

$\{m_{n,t}\}$  - provide symbolic encoding of POs



# 2D Symbolic Dynamics

T	2	4	1	2	1	4	2	1	3	4	2	3	2	3	4	3
	2	3	1	2	1	3	4	2	3	1	2	3	1	4	2	3
	3	2	1	4	3	4	1	1	1	3	1	4	1	2	1	1
	2	4	2	1	4	2	4	3	3	1	2	4	4	1	2	3
	4	1	4	3	2	4	2	1	4	4	1	4	3	4	1	2
	3	4	3	1	3	2	4	2	3	1	2	1	4	2	4	4
	1	1	2	4	3	1	3	2	3	1	4	3	3	4	2	4
	1	4	2	3	1	4	2	4	4	4	3	4	2	2	4	3
	3	2	3	1	4	1	4	1	2	1	2	4	3	1	3	1
	1	4	2	2	3	2	1	3	4	4	2	3	1	4	2	3
	3	1	1	4	1	4	3	2	1	3	2	2	3	3	1	4
N																

$$\mathbb{M}_\Gamma = \begin{pmatrix} m_{1,1} & m_{2,1} & \dots & m_{N,1} \\ m_{1,2} & m_{2,2} & \dots & m_{N,2} \\ \vdots & \vdots & \ddots & \vdots \\ m_{1,T} & m_{2,T} & \dots & m_{N,T} \end{pmatrix}$$

- ✓ **Small alphabet** (does not grow with  $N$ )
  - ✓ **Uniqueness** +  $\Gamma$  can be easily restored from  $\mathbb{M}_\Gamma$
  - ✓ **Locality** ( $r \times r$  square of symbols around  $(n, t)$  defines approx. position of the  $n$ 'th particle at the time  $t$ )
- B.G. V. Osipov (2015),  
 B.G., L Han, R. Jafari, A. K. Saremi, P Cvitanović (2016)

# Example of Partner Orbits

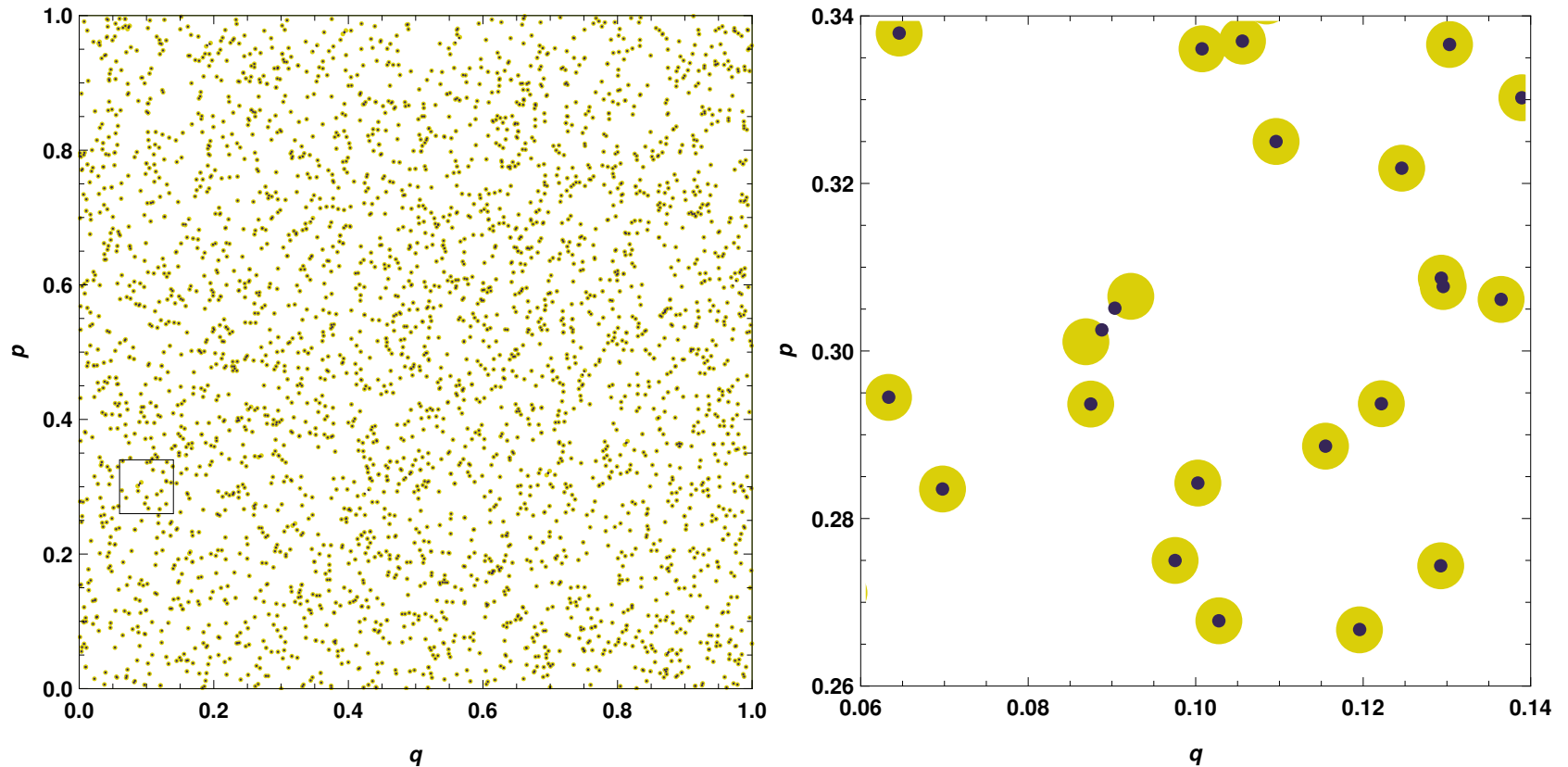
$$T = 50, N = 70, a = 3, b = 2$$

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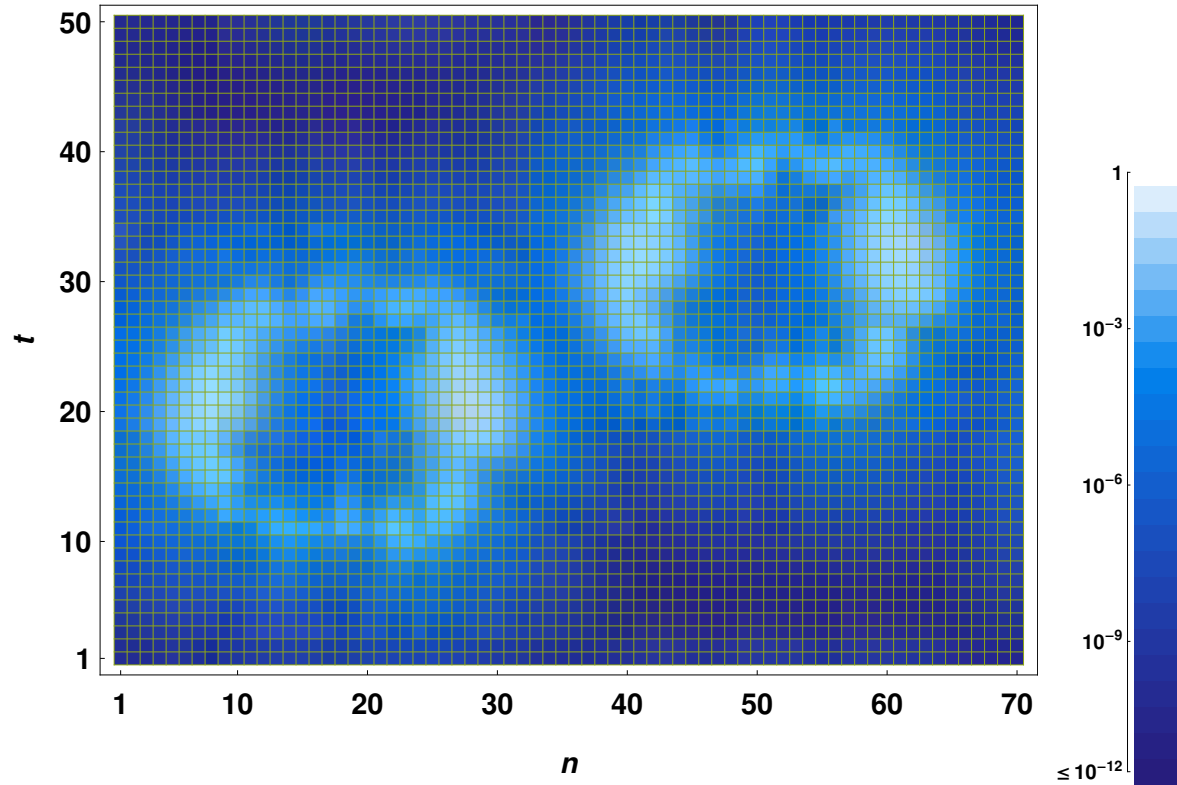
m	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 3 \end{pmatrix}$	$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$	$\begin{pmatrix} 2 \\ 4 \end{pmatrix}$	$\begin{pmatrix} 2 \\ 5 \end{pmatrix}$	$\begin{pmatrix} 3 \\ 5 \end{pmatrix}$	$\begin{pmatrix} 3 \\ 6 \end{pmatrix}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} -1 \\ -1 \end{pmatrix}$	$\begin{pmatrix} -1 \\ -2 \end{pmatrix}$	$\begin{pmatrix} -1 \\ -3 \end{pmatrix}$	$\begin{pmatrix} -2 \\ -3 \end{pmatrix}$	$\begin{pmatrix} -2 \\ -4 \end{pmatrix}$
a	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

# Example of Partner Orbits



**All the points of  $\Gamma = \{(q_{n,t}, p_{n,t})\}$  and  $\bar{\Gamma} = \{(\bar{q}_{n,t}, \bar{p}_{n,t})\}$  are paired**

# Distances between paired points



$$d_{n,t} = \sqrt{(q_{n,t} - \bar{q}_{n',t'})^2 + (p_{n,t} - \bar{p}_{n',t'})^2},$$

**Largest distances  $\sim 2 \cdot 10^{-3}$  are between points in encounters**

# Quantisation

Hannay, Berry (1980); Keating (1991)

$U_N$  is  $L^N \times L^N$  unitary matrix,  $L = \hbar_{eff}^{-1}$

**Translational symmetries:**  $\Rightarrow N$  subspectra  
approximately of the same size  $= L^N / N$

## Gutzwiller trace formula

Rivas, Saraceno, A. de Almeida (2000)

$$\mathrm{Tr} (U_N)^T = \left| \det(\mathcal{B}_N^T - 1) \right|^{-\frac{1}{2}} \sum_{\Gamma \in \mathrm{PO}} \exp(-i2\pi L S_\Gamma).$$

All entries are symmetric under exchange  $N \leftrightarrow T$

# Quantum Duality

$$\mathrm{Tr} (U_N)^T = \mathrm{Tr} (U_T)^N$$

$$\text{Form Factor: } K_N(T) = \frac{1}{2L^N} \left\langle \left| \mathrm{Tr} (U_N)^T \right|^2 \right\rangle$$

For short times  $T < n_E = \lambda^{-1} \log L$ ,  $N \sim L^T$

**Regime dual to universal:**

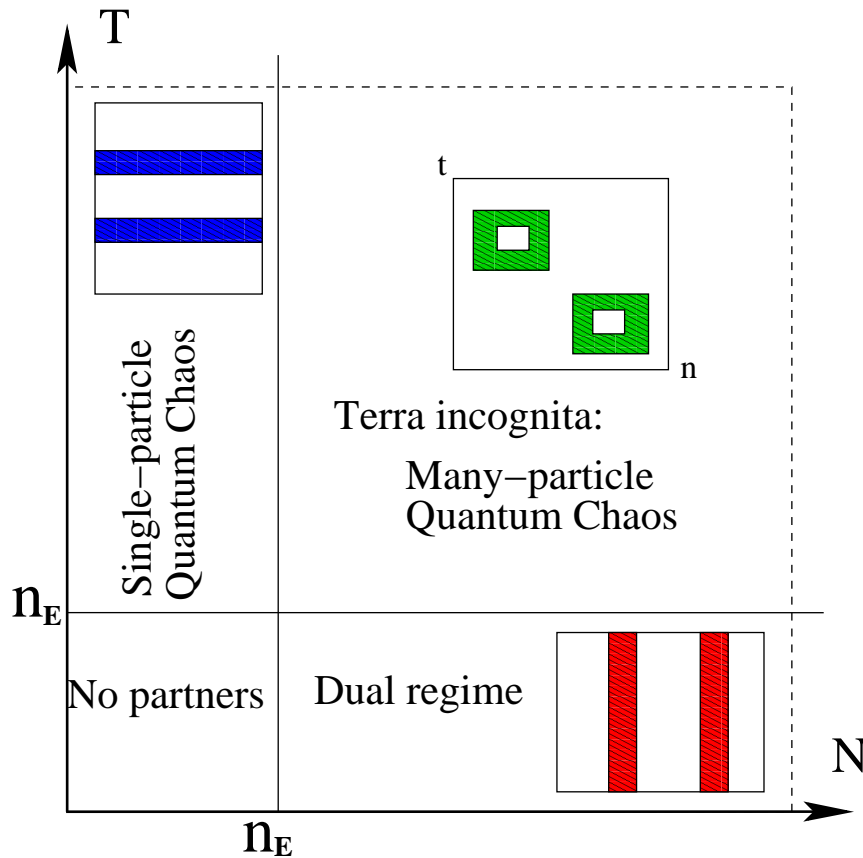
$$K_N(T) = L^{T-N} K_\beta(TN/L^T)$$

In particular for very short times  $L^T/T < N$ ,  $K_\beta \approx 1$

$$K_N(T) \approx L^T / L^N$$

**Short time exponential growth instead of linear  $L^T / L^N$**

# Summary



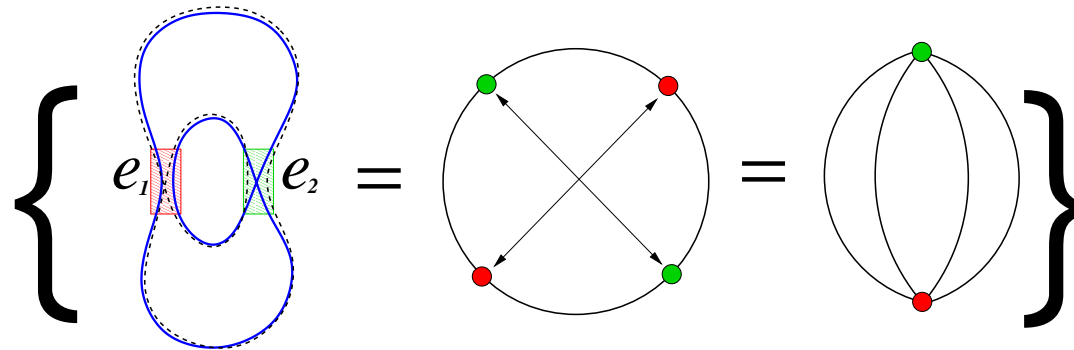
$$\mathcal{K} = \frac{1}{TN} \left\langle \left| \text{Tr} (U_N)^T \right|^2 \right\rangle$$

**Duality:**

$$\mathcal{K}(N, T) = \mathcal{K}(T, N)$$

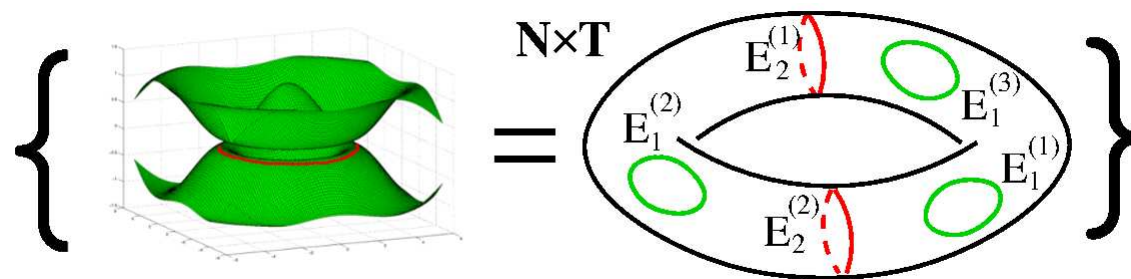
# Many-particle Semiclassical Programm

Single-particle structure diagrams:



Distinguished by order of encounters

Many-particle structure diagrams:



Distinguished by order and winding numbers  $\omega$  of encounters!